

Marsilius of Inghen on Insolubles and the English Tradition

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Abstract

In the course of presenting his own solution to the insolubles (logical paradoxes such as the Liar), Marsilius of Inghen criticises four earlier theories, which appear to be those of Albert of Saxony, (the early) Buridan, Roger Swyneshed and a modification of William Heytesbury's solution which we find in many textbooks and anonymous treatises known as presentations of the *Logica Oxoniensis*. Marsilius's solution bears interesting resemblances to all four, but has its own distinctive features. The core idea of his solution is that all propositions have a two-fold signification, a material signification and a formal one. The material signification, also called the primary or direct signification, is what most would call the proposition's usual signification; e.g., the material signification of 'This proposition is false' is that that proposition is false. Its formal, *aka* indirect or reflexive, signification is, in the case of affirmative propositions, that the subject and predicate supposit for the same thing, and in the case of negative propositions, that they do not. This reflexive signification derives from the meaning of the (affirmative *resp.* negative) copula. Thus the reflexive signification of 'This proposition is false' is that 'this proposition' and 'false' supposit for the same thing, that is, that it is false that that proposition is false. Presenting Marsilius's formal signification in such cases as stating of that proposition's being false, for example (which is the material signification of 'This proposition is false'), that it is false (that is, falls under the supposition of 'false') suggested to Paul Spade that Marsilius's solution was a development of Gregory of Rimini's account. I will argue that any resemblance here is, in the absence of any external evidence, superficial and coincidental, and that Marsilius's view is much closer to the Oxford solutions and Albert's—Albert and Marsilius being, after all, members of the English Nation at Paris. Marsilius's arguments in favour of his theory, and his application of the solution to a range of insolubles, are well worth looking at in detail, which I will do, though not at the length which Marsilius devotes to it.

Marsilius of Inghen's treatise on insolubles is preserved in five manuscripts, two of which contain only incomplete copies. Complete texts exist in Cracow: Jagiellonian Library, 2116, ff.196r-216r (siglum C); Lübeck: Stadtsbibliothek, Phil. 8^o 2, ff.1r-24v (siglum L);¹ and Bibliotheca Apostolica Vaticana Pal.lat.995, ff.63r-89r (siglum S).² Incomplete copies are found in Munich: Bayerische Staatsbibliothek, Codices Latini Monacenses 7709, ff.63r-70v; and Vienna:

¹ The Lübeck manuscript is available online at https://digital-stadtbibliothek.luebeck.de/viewer/image/Msphilos82/9/LOG_0003/.

² The Vatican manuscript is available online at https://digi.vatlib.it/view/MSS_Pal.lat.995.

Österreichische Nationalbibliothek, VPL 5162, ff.171r-174v.³ My knowledge of Marsilius's text is primarily based on the Vatican manuscript, partially cross-checked against the Cracow manuscript. I know of the Munich and Vienna manuscripts only through a preliminary transcription by Bert Bos. The Lübeck manuscript appears to follow the Vatican manuscript closely.

1. Insolubles

First of all, what are the insolubles? They are a certain sort of paralogism or sophism, the paradigmatic example among medieval authors being Socrates saying 'Socrates says a falsehood' (*Sortes dicit falsum*) and nothing else. But there is no straightforward agreement among the medievals as to which paralogisms should be included under the epithet 'insoluble'. There is strong consensus that the term should not be taken literally.⁴ E.g., Marsilius writes (echoing many others):

Let it first be noted that in one way, 'insoluble' is said of what is in no way soluble, and that is not how it is taken here.⁵

It is often conceded, he says, that they can only be solved with difficulty:

Secondly, [an insoluble is so]⁶ called when it is soluble with difficulty, and it is taken in that way here, but not all propositions soluble with difficulty are included

³ Contrary to what is indicated in Bos's list of Marsilius's texts in his *Marsilius of Inghen: Treatises on the Properties of Terms* (Dordrecht: Reidel, 1983), p.25, the *Insolubles* is not to be found in Uppsala: Universitetsbiblioteket, C 640, a copy of which I have seen, and this is confirmed by the description of the manuscript in M. Anderssen-Schnitt and M. Hedlund, *Mittelalterliche Handschriften der Universitätsbibliothek Uppsala: Katalog über die C-Sammlung*, vol.6 (Stockholm: Almqvist & Wiksell International, 1993), pp.190–1.

⁴ See Walter Burley, *Insolubilia* (in Roure, Marie-Louise, "La problématique des propositions insolubles au XIII^e siècle et au début du XIV^e, suivie de l'édition des traités de W. Shyreswood, W. Burleigh et Th. Bradwardine," *Archives d'histoire doctrinale et littéraire du moyen âge* 36–37 (1970), 205–326), p.268, § 2.01; Thomas Bradwardine, *Insolubilia*, ed. and trans. Stephen Read (Leuven: Peeters, 2010), p.62, § 2.1; Paul of Venice, *Logica Magna: the treatise on insolubles*, ed. Stephen Read and Barbara Bartocci (Leuven: Peeters, 2022), p.142, § 2.1.8.1; and many other sources.

⁵ Marsilius, *Insolubilia* ch.1 (S f.63r): "*sit primo notandum quod insolubile uno modo dicitur quasi nullo modo solubile et sic non capitur hic.*" (Latin text and English translations are my own unless otherwise specified. Precise locations are given to the Vatican ms S, although the text has been edited from S, C and L.)

here, but only those whose difficulty [of solution] derives from the signification of the terms.⁷

Even so, there is no agreement even in the extension of the term, let alone how to characterise insolubles. The characterisation often turns on the author's own preferred solution, and that can be seen clearly and dramatically in Paul of Venice's various accounts, where he proposes different solutions in different works. In his *Logica Parva*, written specifically for young beginners (*iuvenes incipientes*),⁸ the solution he offers is what has been dubbed the modified Heytesbury solution, typical of the *Logica Oxoniensis*.⁹ There Paul defines an insoluble as "a proposition signifying consequentially (*assertive significans*) its own falsehood," (see n.10) later distinguishing insolubles unconditionally (*insolubile simpliciter*) from insolubles conditionally (*secundum quid*):

... Some insolubles are insoluble unconditionally, some conditionally. An insoluble unconditionally is one to which a scenario is attached which implies a contradiction

⁶ Square brackets indicate editorial insertions or comments.

⁷ Marsilius, *Insolubilia* ch.1 (*loc.cit.*): "*Secundo dicitur quasi difficulter solubile et sic potest capi hic non tamen quod de omnibus propositionibus difficulter solutionibus hic determinetur, sed solum de quibusdam quarum difficultas provenit ex significatione terminorum.*" See also William Ockham, *Summa Logicae*, ed. Philotheus Boehner, Gedeon Gál and Stephen Brown, *Opera Philosophica* vol. 1 (St Bonaventure: The Franciscan Institute, 1974), III-46, p.744: "*Circa insolubilia sciendum est quod non ideo dicuntur a sophistis aliqua insolubilia quia nullo modo possunt solvi, sed quia cum difficultate solvuntur;*" and Albert of Saxony, *Perutilis Logica* ch.6, in Albert von Sachsen, *Logik*, ed. Harald Berger (Hamburg: Felix Meiner, 2010), p.1100: "*Et dicuntur insolubilia non quia nullo modo possint solvi, sed quia solvere est difficile.*"

⁸ Paul of Venice, *Logica Parva*, ed. Alan R. Perreiah (Leiden: Brill, 2002), p.150, amended against ms BAV, Vat.Lat. 5363, f.39rb: "*Nota quod non quecumque fuit locutus hic, seu in ceteris tractatibus, ego dixi secundum intentionem propriam, sed partim secundum intentionem aliorum, ut iuvenes incipientes proficere facilius introducantur* (Notice that not everything I have said here, or in other treatises, have I said according to my own view, but partly according to the view of others, in order to enable young beginners to progress more easily)." For an alternative translation, see Paul of Venice, *Logica Parva*, trans. Alan R. Perreiah (Munich: Philosophia Verlag, 1984), pp.255–6.

⁹ See, e.g., Spade, Paul Vincent and Stephen Read, "Insolubles", *The Stanford Encyclopedia of Philosophy* (Winter 2021 Edition), Edward N. Zalta (ed.), § 3.5: <https://plato.stanford.edu/archives/win2021/entries/insolubles/#ModHeyt>.

if admitted ... An insoluble conditionally is one to which a scenario is attached which does not imply a contradiction if admitted.¹⁰

This echoes Heytesbury's distinction between mention of a proposition whose truth entails its falsity if the words are taken only in their usual meaning and one where it is allowed that the words are not taken so precisely.¹¹ The first results in contradiction and so should not be admitted, but the second can be admitted without contradiction. Heytesbury infers that in the latter case, the proposition must have some additional meaning, but famously declined to say what that hidden meaning was.¹² The modification made by several anonymous writers, but also by John of Holland, John Hunter and others including Paul is that the additional meaning is a statement of the proposition's own truth.¹³

¹⁰ Paul of Venice, *Logica Parva* (ed. Perreiah, p.128): "*Insolubile est propositio se esse falsam assertive significans ... Insolubiliū aliquod est insolubile simpliciter, aliquod vero secundum quid. Insolubile simpliciter est illud cui annectitur casus quo admissio sequitur contradictio ... Insolubile secundum quid est illud cui casus annectitur quo admissio non sequitur contradictio.*" (For an alternative translation, see *Logica parva*, trans. Perreiah, p.237.) On the notion of 'significare assertive' see L.M. De Rijk, "Semantics in Richard Billingham and Johannes Venator," in *English Logic in Italy in the 14th and 15th Centuries*, ed. Alfonso Maierù (Naples: Bibliopolis, 1982), 167–83, p.175.

¹¹ William Heytesbury, *De Insolubilibus*, ed. in Lorenzo Pozzi, *Il Mentitore e il Medioevo* (Parma: Edizioni Zara, 1987), 201–57, p.238 §§ 3.05–06: "*Ut ergo ad propositum veniamus fiat haec divisio. Si ponitur casus de insolubili aut ponitur qualiter illud insolubile debeat significare aut non. Si non, resondendum est ad illud, proposito illo insolubili, omnino sicut respondetur ad illud non supposito illo casu ... Secundo advertendum est quod, cum ponitur casus de insolubili et cum ho supponitur quod illud insolubile significet praecise sicut termini illius praetendunt [communiter], casus ille nullatenus admittatu.*" Eng.trans. in William Heytesbury, *On Insoluble Sentences*, trans. Paul Vincent Spade (Toronto: Pontifical Institute of Medieval Studies, 1979), pp.47–48 §§ 47–49.

¹² In a similar vein, Alonzo Church, 'Review of Alexander Koyré, "The liar",' *Journal of Symbolic Logic* 11 (1946), inferred from Epimenides's reported claim that all Cretans are liars that there must have been another Cretan utterance, indeed, a true one. Church's observation was picked up and developed by Arthur Prior: see his 'Epimenides the Cretan,' *Journal of Symbolic Logic* 23 (1958), 261–6, p.261.

¹³ See Stephen Read, 'Theories of paradox from Thomas Bradwardine to Paul of Venice', in Stephen Read and Barbara Bartocci (edd.), *Theories of Paradox in the Middle Ages* (London: College Publications, 2023), 11–42, §4. On whether Hunter (*aka* Venator) is the author of the treatise attributed to him, see Fabienne Pironet, "William Heytesbury and the treatment of Insolubilia in 14th-century England", in Rahman, Shahid, Tero Tulenheimo, and Emmanuel Genot (edd.), *Unity, Truth and the Liar: The*

Paul (loc.cit.) gives us an example, in which it is assumed that the proposition 'No proposition is true' is the only proposition and signifies only as the words explicitly permit. As we noted, a contradiction follows, so the scenario must either be rejected, or be amended to admit a further, hidden, meaning. In the latter case 'No proposition is true' is an insoluble and signifies both what follows consequentially from the meanings of its terms (*significans assertive*—hence that it is not true, given that it is the only utterance) and that it is true. Consequently, the scenario where it is the only proposition is admissible and 'No proposition is true' is simply false in being self-contradictory.

In his treatise on insolubles in his *Logica Magna*, however, Paul adopts a different solution, namely, that of Roger Swyneshed, and defines insolubles rather differently. Indeed, the definition comes in two forms, a narrower and a broader one. Paul gives the narrower one in the second chapter of the treatise:

An insoluble proposition is a proposition having reflection on itself wholly or partially implying its own falsity or that it is not itself true.¹⁴

In brief, insolubles are self-falsifying propositions. A similar definition is adopted by Peter of Ailly. Peter infers that a necessary condition for a proposition's being an insoluble is that it contain a term signifying a proposition, such as 'true', 'false', 'doubtful', 'believed' and so on.¹⁵

Modern Relevance of Medieval Solutions to the Liar Paradox (Berlin: Springer-Verlag, 2008), 255–333, pp.265–6.

¹⁴ Paul of Venice, *Logica Magna: the treatise on insolubles* (ed.cit.), § 2.1.8: "*Propositio insolubilis est propositio habens supra se reflexionem sue falsitatis aut se non esse veram, totaliter vel partialiter illativa.*"

¹⁵ See Peter of Ailly, *Insolubilia*, in Markus Erne, *Mentale Sätze und das Problem semantischer Antinomien: Die Insolubilia von Pierre d'Ailly. Historische Studie und textkritische Edition* (Turnhout: Brepols, 2022), pp.76–7: "*Tertia descriptio est huius termini 'propositio insolubilis' seu 'insolubile'. Unde dico quod est propositio quae significat se esse falsam ... Ex praedictis sequuntur aliqua corollaria. Primo sequitur ista regula notabilis: Quod nulla propositio habet reflexionem supra se, nisi in qua ponitur terminus appropriate significans propositionem, sicut sunt tales termini: 'verum', 'falsum', 'universale', 'particulare', 'affirmativum', 'negativum', 'concedendum', 'negandum', 'dubium', 'certum' et similes* (The third definition is of the term 'insoluble proposition' or 'insoluble'. For I say that it is a proposition which signifies itself to be false ... From this there follow some corollaries: first, this important rule follows, that no proposition has reflection on itself unless there occurs in it a term appropriately

Both Peter and Paul comment that their definition excludes many propositions counted as insolubles by others, such as 'Socrates will not cross the bridge' and 'Plato will not have a penny', for he says, they do not have reflection on themselves (*op.cit.*, § 2.1.8.3). Indeed, Peter criticises Marsilius's account of insolubles for just this reason. But Paul is not consistent here, for in the fifth chapter he includes them under what he calls "insolubles that don't appear at first glance to be insolubles" (*insolubilia que prima facie insolubilia non apparent*). It is in the eighth chapter that he comes to further cases that he believes only appear to be insolubles, such as 'This proposition is not known to you' and 'This is in doubt for you', which others would include as epistemic insolubles.¹⁶

Swyneshed had himself given a broader definition which included these epistemic paradoxes:

An insoluble as put forward is a proposition signifying principally as things are or other than things are which is relevant to inferring itself to be false or unknown or [not] believed, and so on.¹⁷

Paul himself is tempted to broaden his definition to include the epistemic insolubles, when, for example, he presents the fourth Conclusion in the *Logica Magna*:

There is a formally sound consecution, known by you to be so, signifying [exactly] by the composition of its parts, where the premise is known by you, yet the conclusion is not known by you.¹⁸

signifying a proposition, such as the terms 'true', 'false', 'universal', 'partial', 'affirmative', 'negative', 'granted', 'denied', 'doubtful', 'certain', and such like)." For an alternative translation, see Paul V. Spade, *Peter of Ailly: Concepts and Insolubles* (Dordrecht, Boston and London: Reidel, 1980), pp.64–5.

¹⁶ See, e.g., Bradwardine, *Insolubilia* (ed.cit.), ch.9; and Swyneshed, *Insolubilia*, in Spade, Paul Vincent, "Roger Swyneshed's *Obligationes*: edition and comments," *Archives d'histoire doctrinale et littéraire du moyen âge* 44 (1977), 243–85, § IV.

¹⁷ Swyneshed, *Insolubilia* (ed.cit.), § 16: "*Insolubile ad propositum est propositio significans principaliter sicut est vel aliter quam est pertinens ad inferendum se ipsam fore falsam vel nescitam vel [non] creditam, et sic de singulis.*" ('non' is added for sense in that last clause following the edition in Pozzi, *Il Mentitore e il Medioevo*, p.282.

¹⁸ Paul of Venice, *Logica Magna: the treatise on insolubles* (ed. cit.), § 2.3.4: "*Quarta conclusio: aliqua consequentia est bona et formalis, scita a te esse talis, significans*

The example he gives is what may be called the Inferential Knower Paradox:¹⁹

This is unknown to you, therefore this is unknown to you,

where each occurrence of 'this' refers to the conclusion. For, he says,

... the premise is known by you, because you know that the conclusion is not known, since it is an insoluble that implies that it itself is unknown.²⁰

Thus the idea in the broadening of the definition is to say that just as propositions which imply their own falsehood are self-falsifying and so are false, so too propositions which imply they are not known are not known and those which imply they are not believed are not believed, and so on.

2. Marsilius's Critique of Other Solutions

Marsilius's definition of the insolubles characterises them as those propositions which signify themselves to be false by means of their reflection on themselves:

Now I call a proposition having reflection on itself by means of which it signifies itself to be false, an insoluble or self-falsifying proposition.²¹

Having characterised insolubles this way, Marsilius proceeds immediately to consider four other opinions before turning to his own solution:

[adequate] ex compositione suarum partium, et antecedens est scitum a te et consequens non est scitum a te." I will use 'consequence' or 'valid inference' to translate '*consequentia*' when it is the defining property that is in question, and 'consecution' when a particular sequence of premises and conclusion, valid or not, is being discussed. I will translate '*bona*' as 'sound' in this context, but it should be understood that 'sound' (*bona*) and 'valid' (*valet*) are equivalent.

¹⁹ On the Knower Paradox ('This proposition is not known', referring to itself), see, e.g., Roy Sorenson, 'Epistemic Paradoxes', *The Stanford Encyclopedia of Philosophy* (Fall 2020 Edition), Edward N. Zalta (ed.), § 5.1:

<https://plato.stanford.edu/archives/win2021/entries/epistemic-paradoxes/#KnoPar>.

Marsilius discusses the Knower Paradox at the end of ch.6 (S 85r): "*Aliud sophisma et ultimum illius capituli sit tale: a non est scitum, et sit a nomen proprium illius sophismatis.*"

²⁰ Paul of Venice, *Logica Magna: the treatise on insolubles* (ed.cit.), § 2.3.4: "... *antecedens est scitum a te, quia scis illud consequens non sciri, cum sit insolubile asserens se nesciri.*"

²¹ Marsilius, *Insolubilia* ch.2 (S f.70r): "*Voco autem propositionem insolubilem seu falsificantem se propositionem habentem reflexionem super se mediante qua significat se esse falsam.*"

As regards the second chapter, it must be seen why a self-falsifying proposition is said to be false. To that end, first, the solutions of others will be presented and disproved, and secondly, the true solution will be presented.²²

His choice of other solutions to describe and set aside is interesting, for they each give a different reason why self-falsifying propositions are said to be false. The second solution is attributed to Buridan (by Marsilius or by the scribe) in the Vatican manuscript, in an addition between the lines apparently by the same scribe who penned the whole manuscript:

In another way, a second solution, that of many authoritative people [indeed, of Buridan], claims that a self-falsifying proposition like ‘Socrates says a falsehood’ is false because it signifies itself to be true and false and that is not so.²³

This is not Buridan’s well-known later solution, for that solution denies that self-falsifying propositions signify themselves to be true, claiming, rather, that they are inconsistent and false because every proposition “has the power to imply” its own truth:

Therefore, we put this otherwise, in a manner closer to the truth, namely, that every proposition has the power to imply [*implicat virtualiter*] another proposition in which the predicate ‘true’ is affirmed of the subject that supposits for [the original proposition]; and I say ‘has the power to imply’ in the sense in which what is antecedent implies what follows from it.²⁴

²² Marsilius, *loc.cit.*: “Quantum ad secundum [capitulum] videndum est unde propositio falsificans se dicatur falsa. Circa quod primo ponende sunt opiniones aliorum et improbande, secundo ponenda est opinio vera.”

²³ S, f.70^v ll.9–11: “Aliter dicit secunda opinio multorum valencium [scilicet buridani add. inter linea] quod propositio falsificans se sicut: sortes dicit falsum, est falsa quia significat se esse veram et falsam et sic non est.”

²⁴ See John Buridan, *Summulae de Practica Sophismatum*, ed. Fabienne Pironet (Turnhout: Brepols, 2004), 155: “Ideo dicitur aliter, propinquius veritati, scilicet quod quaelibet propositio implicat virtualiter aliam propositionem qua de subiecto pro ea supponente affirmaretur hoc praedicatum ‘verum’; dico ‘implicat virtualiter’ sicut antecedens implicat illud quod ad ipsum sequitur.” Klima’s translation, in John Buridan, *Summulae de Dialectica*, trans. Gyula Klima (New Haven: Yale University Press, 2001), 969, like Scott’s, in John Buridan, *Sophisms on Meaning and Truth*, Eng.trans. T.K. Scott (New York: Appleton-Century-Crofts 1966), 195 and Hughes’s, in *John Buridan on Self-Reference* (Cambridge: Cambridge University Press 1982), 49,

Instead, what is described in the second solution that Marsilius considers may be the position which Buridan seems to have maintained in his early writings, such as his *Questiones Elencorum* and *Quaestiones in Analytica Priora*:

I grant that the proposition 'I say something false' is false. Nonetheless, things are as it signifies, because it signifies itself to be false and that is how things are. But things are not altogether as it signifies, because by its form every proposition asserts and means that it itself is true, even though by the signification of its terms it can at the same time mean that it itself is false. Because it not only signifies itself to be false, but also, by the general condition of a proposition, signifies itself to be true, and it is not true. For this reason, even though it signifies partly in that way, nonetheless, things are not wholly or altogether like that. For that reason it is false.²⁵

Marsilius rejects this solution for the same reason he rejects the first solution:

But this response is no more valid than the preceding one, because it is based on assuming that this consecution is valid:

The subject and predicate of this affirmative proposition supposit for the same, therefore it is true,

and it is certain that it is not true, as was said in the disproof of the preceding account.²⁶

renders 'implicat virtualiter' as 'virtually implies', which is at best unhelpful and at worst misleading, suggesting it almost implies it but somehow fails. See further n.29 below.

²⁵ John Buridan, *Quaestiones in duos libros Aristotelis Analyticorum Posteriorum*, ed. Hubert Hubien (unpublished), Q 10: "[de ista propositione] 'ego dico falsum', concedo quod est falsa. Tamen sicut ipsa significat esse ita est, quia significat se esse falsam et ita est. Sed tamen non qualitercumque ipsa significat esse ita est, quoniam omnis propositio de eius forma asserit et designat se esse ueram, licet ex significatione terminorum possit cum hoc designare se esse falsam, quia ipsa non solum significat se esse falsam, sed etiam, ex communi condicione propositionis, significat se esse ueram, et non est uera. Ideo licet aliquantulum sic significet, non tamen totaliter uel qualitercumque ita est. Ideo est falsa." See also Buridan, *Quaestiones Elencorum*, ed. Ria van der Lecq and Henk A.G. Braakhuis (Nijmegen: Ingenium, 1994), Q 19, and Buridan, *Summulae de Dialectica* (trans. Klima, 1967), where he says: "... it seemed to me elsewhere that ... every proposition by its form signifies or asserts itself to be true" (ed. Pironet, *Summulae de Practica Sophismatum*, 154: ... ita visum fuit in aliis, quod ... omnis propositio de forma sua significat vel asserit se esse ueram).

²⁶ Marsilius, *Insolubilia*, ch.2 (S f.70v): "Sed hec responsio non plus ualet quam precedens, quia fundatur super hoc quod hec consequentia ualeat: subiectum et

It is no surprise that Marsilius's objection to the second solution is the same as his rejection of the first, since the two solutions sound very similar and are supported by what seems to be the same argument:

The first solution is that a self-falsifying proposition is said to be false because it signifies [things to be] in a way in which they are not, and they prove it like this: because it signifies itself to be false by hypothesis and it signifies itself to be true, but that is not so.²⁷

The claim made by both solutions that self-falsifying propositions signify themselves to be true follows from the claim that every proposition signifies itself to be true. This claim is supported by the following argument:

... because every proposition is affirmative or negative. If affirmative, it signifies that its subject and predicate supposit for the same thing, therefore, it signifies itself to be true; while if negative, it signifies that its subject and predicate do not supposit for the same thing, and thus again it signifies itself to be true.²⁸

Essentially the same argument is presented in support of the second solution:

But that ['Socrates says a falsehood'] signifies itself to be true is proved because from its signifying the subject and predicate to supposit for the same, with its existential premise, it follows that it is true, therefore, it signifies itself to be true. The premise is proved because, given that this proposition 'Socrates says a falsehood' exists and signifies its subject and predicate to supposit for the same, and the proposition exists, it follows that it signifies itself to be true.²⁹

predicatum huius propositionis affirmative supponunt pro eodem, ergo est vera, et certum est quod istud non est verum, ut dictum est in precedentis modo in improbatione."

²⁷ Marsilius, *Insolublia*, ch.2 (S f.70r): "*Et est prima opinio quod propositio falsificans se ex eo dicitur falsa quia significat aliquantulum qualiter non est et probant sic quia significat se esse falsam ex casu et significat se esse veram modo sic non est.*"

²⁸ Marsilius, *loc.cit.*: "*...probant quia vel omnis propositio est affirmativa vel negativa. Si affirmativa significat esse idem pro quo subiectum et predicatum eius supponunt, ergo significat se esse veram. Si autem negativa significat non esse idem pro quo supponunt subiectum et predicatum eius et sic iterum significat se esse veram.*"

²⁹ Marsilius, *Insolublia*, ch.2 (S f.70v): "*Quod autem significat se esse veram probatur quia ex eo significare subiectum et predicatum supponere pro eodem cum constantia sui sequitur se esse veram ergo se significare esse veram. Antecedens probatur quia sequitur illa propositio: sortes dicit falsum, est et significat suum subiectum et*

This second solution is thus seen to be a strange combination of Buridan's early view (that, like all propositions, insolubles signify themselves to be true) with his later qualification that an existential premise (*constantia*) is needed:³⁰

But although this solution is in my opinion closer to the truth, it is still not perfect, for it assumes something false, namely, that any proposition implies such a consequent. For assuming that the proposition 'A horse runs' is named by the proper name 'B', then this is not valid: 'A horse runs; therefore, B is true', as has been said in connection with the second sophism of this chapter. Therefore, perfecting this solution, we have to say that every proposition, adding that it exists, implies that it is true.³¹

But the first solution is also reminiscent of Albert of Saxony's solution, and indeed, like Buridan in the *Questiones Elencorum* (ed.cit., § 19.3.2, p.92), Albert supports his claim that every proposition signifies its own truth by the argument that Marsilius rejects:

... the first assumption: Every proposition is affirmative or negative. Second assumption: For any affirmative proposition to be true is for its subject and predicate to supposit for the same thing (and vice versa); and for it to be false is for its subject and predicate to supposit for what is not the same (and vice versa). Third assumption: For any negative proposition to be true is for its subject and predicate to supposit for what is not the same (and vice versa); and for it to be false is for its subject and predicate to supposit for the same thing [and vice versa]. Fourth assumption: Every affirmative proposition signifies that what its subject and

predicatum supponere pro eodem et propositio est, ergo significat se esse veram." The at first strange addition of the extra premise, 'and the proposition exists', perhaps explains that for the conclusion to follow not only must 'Socrates says a falsehood' exist but also 'That Socrates says a falsehood is true' must exist too.

³⁰ On the notion of '*constantia*', see, e.g., Ashworth, 'Existential assumptions in late medieval logic', *American Philosophical Quarterly* 10 (1973), 141–7.

³¹ Buridan, *Sophismata*, in *Summulae de Dialectica*, trans. Klima, 970; ed. Pironet, *Summulae de Practica Sophismatum*, 156: "*Sed haec solutio quamvis, ut puto, sit propinqua veritati, tamen adhuc non est perfecta, quia supponit falsum, scilicet quod ad quamlibet propositionem sequatur tale consequens. Dato enim quod haec propositio 'equus currit' vocetur nomine proprio B, tunc non sequitur 'equus currit; ergo B est vera', sicut dicebatur in secundo sophismate huius capituli. Et ideo, perficiendo istam solutionem, debemus dicere quod ad omnem propositionem, cum appositione quod ipsa est, sequitur quod ipsa est vera.*"

predicate supposit for is the same, and the affirmed copula in it shows this clearly. Fifth assumption: Every negative proposition signifies that what its subject and predicate supposit for is not the same, and the negated copula in it shows this clearly. Sixth assumption: It is impossible that the same proposition be true and false at the same time.³²

Albert puts these assumptions together in support of his third thesis (*conclusio*):

Every proposition in the world signifies that it is true.³³

Marsilius has a powerful objection to this argument:

But this mode [of reasoning] is inadequate. For this consecution is not valid:

This proposition signifies that its subject and predicate supposit for the same thing, therefore it signifies itself to be true,

just as indeed:

The subject and predicate of this affirmative proposition supposit for the same thing, therefore it is true

Is not valid, because there is a counter-example in self-falsifying propositions.

Thus:

The subject and predicate supposit for the same thing, therefore it signifies itself to be true

³² Albert of Saxony, *Insolubles*, Eng. trans. in *The Cambridge Translations of Medieval Philosophical Texts*, vol.1: *Logic and the Philosophy of Language*, ed. N. Kretzmann and E. Stump (Cambridge 1988), 339; ed. Berger, *Logik*, 1100–02: "... *prima suppositio: Omnis propositio vel est affirmativa vel negativa. Secunda suppositio: Omnem propositionem affirmativam esse veram est idem esse, pro quo supponit eius subiectum et praedicatum, et e converso. Et ipsam esse falsam est non idem esse, pro quo supponit eius subiectum et praedicatum, et e converso. Tertia suppositio: Omnem propositionem negativam esse veram est non esse idem, pro quo supponit eius subiectum et praedicatum, et e converso. Et ipsam esse falsam est esse idem, pro quo supponit eius subiectum et praedicatum, et e converso. Quarta suppositio: Omnis propositio affirmativa significat idem esse, pro quo supponit eius subiectum et praedicatum, et hoc ostendit nobis manifeste copula in ea affirmata. Quinta suppositio: Omnis propositio negativa significat non idem esse, pro quo supponit eius subiectum et praedicatum, et hoc ostendit nobis manifeste copula in ea negata. Sexta suppositio: Impossibile est eandem propositionem esse veram et falsam.*"

³³ Albert, trans. Kretzmann and Stump, 340; ed. Berger, 1104: "*Omnis propositio mundi significat se esse veram.*"

is not valid.³⁴

The objection is a clever ad hominem argument. Albert and early Buridan both believe that insolubles signify that what its subject and predicate supposit for is the same (see Albert's fourth and fifth assumptions above) and that insolubles are false, so the fact that its subject and predicate supposit for the same thing cannot suffice for its truth. For Albert accepts that

A true proposition is one such that things are however it signifies they are.³⁵

Indeed, that was Albert's first definition (*descriptio*). So his second and third assumptions must be mistaken, and his proof of his third thesis, that every proposition signifies that it is true, fails. As Marsilius says:

This is confirmed because it is not clear in what way this proposition 'Socrates is a man' signifies itself to be true, because it does not signify this directly, as is familiar, nor does it signify this consequentially, as was already argued, because:

It signifies that the subject and predicate supposit for the same thing, therefore, it signifies itself to be true,

is in no way valid, because the subject and predicate suppositing for the same thing does not suffice for this, that an affirmative subject-predicate proposition is said to be true, as was said.³⁶

³⁴ Marsilius, *Insolubilia*, ch.2 (S f.70v): "*Sed ille modus non est sufficiens. Nam hec consequentia non valet: hec propositio significat idem esse pro quo supponunt subiectum et predicatum eius, ergo significat se esse veram, sicut enim non sequitur: idem est pro quo subiectum et predicatum huius propositionis affirmative supponunt, igitur est vera, quia instantia patitur in propositionibus falsificantibus se. Sic non sequitur: significat idem esse pro quo supponunt subiectum et predicatum igitur significat se esse veram.*"

³⁵ Albert of Saxony, *Insolubles*, trans. Kretzmann and Stump, 338; ed. Berger, *Logik*, 1100. Buridan states the connection between suppositing for the same thing and truth directly in his *Questiones Elencorum* (ed.cit.), 92: "... for an affirmative proposition signifies the subject and predicate to supposit for the same thing. And this is for it to be true, or at least from the affirmative it follows that it is true ... Secondly, the same is clear concerning a negative proposition ... (... *nam propositio affirmativa significat subiectum et predicatum supponere pro eodem. Et hoc est ipsam esse veram, vel saltem ad affirmativam sequitur ipsam esse veram ... Secundo patet hoc de negativa ...*)."

³⁶ Marsilius, *Insolubilia*, ch.2 (loc.cit.): "*Confirmatur quia non videtur quomodo hec propositio: sortes est homo significat se esse veram quia directe hoc non significat, ut*

Nonetheless, the idea that every proposition signifies, if affirmative, that its subject and predicate supposit for the same thing (and if negative, that they do not) is central to Marsilius's solution to the insolubles. But he denies that it follows that every proposition signifies itself to be true. Insolubles are themselves a counter-example to that claim.

The adherents of the fourth solution rejected by Marsilius are also difficult to identify. It may of course be the case that one or more of these solutions considered are straw men invented by Marsilius for purposes of exposition in leading his readers towards the true solution which is to come. However, it is more likely that they are indeed solutions actually canvassed by Marsilius's contemporaries and predecessors, but perhaps seen through a particular lens for those expository ends, as is so common in other texts of the period. The third solution is, nonetheless, very clearly that of Roger Swyneshed:

The third solution says that a proposition of this sort and any similar one is said to be false only because it reflects its falsity on itself, signifying itself to be false. And if it is argued like this: things are precisely as it signifies, therefore, it is true, they grant the premise and deny the consecution when 'therefore, it is true' is inferred. And regarding the definition of a true proposition, they say that it [should be] understood [only] as regards propositions not having reflection of falsity on themselves.³⁷

This agrees closely with Swyneshed's proposed re-definition of truth and falsity:

notum est, nec significat hoc consecutive, ut iam argutum est, quia nullomodo sequitur: significat subiectum et predicatum supponere pro eodem ergo significat se esse veram, quia subiectum et predicatum supponere pro eodem non sufficit ad hoc quod propositio affirmativa categorica dicitur esse vera, ut dictum est."

³⁷ Marsilius, *loc. cit.*: "*Tertia opinio dicit quod huiusmodi propositio solum ex eo dicitur falsa et quevis consimilis, quia reflectit suam falsitatem supra se, significando se esse falsam. Et si sic [istis mss] arguitur: precise est sic sicut ista significat, igitur est vera, concedunt illam, scilicet antecedens et negant consequentiam cum infertur, ergo est vera. Et ad diffinitionem propositionis vere dicunt quod intelligitur de propositionibus non habentibus reflexionem falsitatis supra se.*"

... a true proposition is a proposition not falsifying itself signifying principally³⁸ as things are either naturally or by imposition whereby it was last imposed to signify ... a false proposition is an utterance falsifying itself or an utterance not falsifying itself signifying principally other than things are either naturally or by the imposition or impositions whereby it was last imposed to signify.³⁹

That is, a proposition which signifies as things are may still be false because it falsifies itself, that is, implies its own falsehood. Including Swyneshed's solution here accords with Marsilius's expository purposes since it illustrates another way, at least by Swyneshed's lights, in which what is normally taken as grounds for a proposition's truth may still fail to suffice for that conclusion, as recorded in the quotation from Marsilius above. In fact, Marsilius rejects this solution on the grounds, *inter alia*, that it "denies the accepted definition of truth and falsehood."⁴⁰ Falsehood is simply a failure of agreement of the thing signified with what is meant, as Aristotle explained in the *Metaphysics*.⁴¹

Returning to the fourth solution which Marsilius considers and rejects, once again, like the first and second, it claims that insolubles signify their own truth:

The fourth solution says that propositions of this kind are said to be false because they signify themselves to be false, by hypothesis, and also signify themselves to be true.

³⁸ Swyneshed uses the phrase 'signifies principally' repeatedly, but seems to mean no more by it than simply 'signifies', since he rejects the claim made by Bradwardine, Heytesbury, Albert, Buridan and others that insolubles, or even all propositions, signify more than appears at first glance. Swyneshed's aim was to solve the insolubles without recourse to any additional signification such as those authors proposed.

³⁹ Swyneshed, *Insolubilia* (ed. cit.), §§ 14–15: "*Propositio vera est propositio non falsificans se principaliter sicut est significans naturaliter aut ex impositione vel impositionibus qua vel quibus ultimo fuit imposita ad significandum ... Propositio falsa est oratio falsificans se vel oratio non falsificans se principaliter aliter quam est significans naturaliter, ex impositione, vel impositionibus qua vel quibus ultimo fuit imposita ad significandum.*"

⁴⁰ Marsilius, *loc.cit.*: "... *interimit descriptiones communes propositionum vere et false prius positas et ab aliis concessis.*"

⁴¹ Marsilius, *loc.cit.*: "*Secundo quia intellectus non capit propositionem affirmativam de presenti et de inesse esse falsam nisi quia propositio non est adequatio rei significate ad intellectum sexto methaphysice et per consequens non aliquo modo significat qualiter non est et huius oppositum hec opinio ponit.*" The reference is to Aristotle, *Metaphysics* E 4.

They prove this because it follows from them along with a true premise that they are true, therefore, they signify themselves to be true. The consecution holds, and the premise is demonstrated because this consecution is valid:

Socrates says a falsehood, and this proposition ‘Socrates says a falsehood’ signifies in that way, therefore, it is true.⁴²

This seems to be a cryptic allusion to the modified Heytesbury solution mentioned in § 1. For what is distinctive of Heytesbury’s solution and its modification as presented in, for example, John of Holland’s *Insolubilia*, is whether or not the term ‘precisely’ is applied to the specification of the insoluble in the scenario. John of Holland considers an objection to his claim that every insoluble is false.

When it is argued, [he says]: ‘This is false, therefore, things are other than it signifies’, the consecution is granted. And further, when it is argued: ‘Things are other than it signifies, and it signifies that Socrates says a falsehood, therefore, things are other than that Socrates says a falsehood’, the consecution is denied. Because it is necessary to add ‘precisely’ to ‘signifies’ in the minor premise ... and when it is argued in this way, the minor premise is both false and irrelevant.⁴³

The revised minor premise is false and so fails to support the unwanted conclusion because insolubles signify more than one thing conjunctively:

⁴² Marsilius, *Insolubilia*, ch.2 (S ff.70v–71r): “*Quarta opinio dicit huiusmodi propositiones dicuntur false quia significant se esse falsas per casum et etiam significant se esse veras, quod probant quia ex eis cum vera assumpta sequitur ipsas esse veras igitur significant se esse veras. Consequentia tenet, antecedens declaratur quia sequitur: sortes dicit falsum et hec propositio: sortes dicit falsum, significat sic, igitur ipsa est vera.*”

⁴³ John of Holland, *Insolubilia*, in John of Holland, *Four Tracts on Logic (Suppositiones, Fallacie, Obligationes, Insolubilia)*, ed. E.P. Bos (Nijmegen: Ingenium, 1985), 129: “*Quando arguitur: ‘hec est falsa, igitur aliter significat quam est’, conceditur consequentia. Et ultra, quando arguitur: ‘hec significat aliter quam est, et hec significat quod Sortes dicit falsum, ergo aliter est quam quod Sortes dicit falsum’, negatur consequentia. Quia oportet addi in minore li precise ad li significat ... et si sic argueretur, negatur minor tamquam falsa et impertinens.*”

The reason is that the insoluble ['Socrates says a falsehood'] signifies conjunctively that Socrates says a falsehood and that 'Socrates says a falsehood' is true (at least, so many say).⁴⁴

If insolubles signified only (that is, precisely) what they immediately appear to signify, contradiction would result. Hence they signify more, as Heytesbury realised. But Holland and others go further than Heytesbury was willing to go: what insolubles signify in addition is their own truth, as the fourth solution which Marsilius considers says.

Marsilius dismisses the fourth opinion as based on a false assumption, namely, that insolubles only signify what they immediately appear to signify:

But this way [of arguing] is not valid, for the consecution [that is, "Socrates says a falsehood, and this proposition 'Socrates says a falsehood' signifies in that way, therefore, it is true"] is only valid if 'precisely' is added to the second premise, and if it were added, the second premise would be false when taken for what falsifies itself, because since it is false it is necessary that it signifies something other than that it itself is false.⁴⁵

Moreover, even if the reasoning is corrected by adding that qualification to the second premise, it still fails to justify the conclusion that such propositions signify their own truth:

Secondly, because even if the consecution were sound: still one may ask whether a self-falsifying proposition is assumed in the first premise, and if so, the premise is false, and so it is no surprise that a false conclusion follows from it.⁴⁶

⁴⁴ Holland, *Insolubilia*, 131: "*Et ratio est: ex quo insolubile significat copulative quod Sortes dicit falsum et quod hec est vera: Sortes dicit falsum (saltem ut plures dicunt).*"

⁴⁵ Marsilius, *Insolubilia*, ch.2 (S f.71r): "*Sed iste modus non valet, non enim valet consequentia nisi adderetur in secunda parte antecedentis: precise, quod si adderetur, antecedens pro secunda parte esset falsum accipiendo illam que se falsificat quia cum illa sit falsa oportet quod aliquid aliud significat quam se esse falsam.*"

⁴⁶ Marsilius, *loc.cit.*: "*Secundo quia esto quod consequentia esset bona: adhuc quereretur utrum in antecedente assumeretur propositio significans se esse falsam in prima parte, et si sic antecedens est falsum, ergo non mirum quod consequens falsum sequitur ex eo.*"

What we can note for future reference, however, is that, apart from the possible inclusion of Buridan, the alternative solutions discussed and dismissed by Marsilius are those of the English school, of Swyneshed, Heytesbury, Holland and the *Logica Oxoniensis*, and of Albert of Saxony, who belonged to the English Nation at Paris.

3. Formal and Material Signification

Solutions to the insolubles in the fourteenth century divide between those which depend on a claim that insolubles, or perhaps all propositions, have an additional signification in addition to what they obviously signify in virtue of their constituent terms, and those which reject any such hidden signification. Proponents of the former kind include Thomas Bradwardine, William Heytesbury, the early Buridan, Albert of Saxony, Gregory of Rimini and many of their successors; proponents of the latter kind of solution include Roger Swyneshed, John Dumbleton and Richard Brinkley. Marsilius belongs to the first camp, but goes beyond his predecessors, such as Bradwardine and Heytesbury, who explicitly attributed an additional signification only to insolubles: he argues, like Albert and Buridan, that all propositions have a dual signification, a material signification and a formal one, respectively called the primary or direct signification and the indirect, connotative or reflexive signification. Then a present-tense non-modal subject-predicate proposition is true if things are wholly as it signifies:

Third definition: affirmative entirely assertoric present-tense propositions are said to be true because things are as they signify according to their total signification. And on the same ground their negative opposite is said to be false. For in all cases the ground of the truth of an affirmative proposition and of the falsity of its negative opposite is the same, and so too of the falsity of the affirmative and of the truth of the negative.⁴⁷

⁴⁷ Marsilius, *Insolubilia*, ch.1 (S f.63v): “*Tunc sit tertia descriptio: propositiones affirmative de presenti mere de inesse ex eo vera dicuntur quia qualitercumque pronunc per eas secundum eius totalem significationem significatur ita est. Et eadem de causa negativa sibi opposita dicitur falsa. In omnibus enim eadem est causa veritatis affirmative et falsitatis negative sibi opposite et falsitatis affirmative et veritatis negative.*”

Marsilius proceeds to adapt this definition for past- and future-tense and modal propositions, and then announces:

A proposition which doesn't exist is neither true nor false. That is clear because if it's true or false then it exists.⁴⁸

Consequently, a proposition cannot be true unless its total signification is compatible with its existing—so, e.g., 'Every proposition is particular' can't be true, even though its contradictory (that is, 'Some proposition is not particular') isn't necessary.⁴⁹

But what are these material and formal significations (or senses or representations—he uses all three terms)? The formal signification is introduced by the fourth and fifth assumptions:

Every affirmative subject-predicate proposition affirms that its subject and predicate supposit for the same thing, or claims that what the extremes are taken for is the same. This is clear by the meaning of the affirmative copula ...

Every negative subject-predicate proposition signifies that the subject does not supposit for what the predicate [supposits] for, i.e., that of which the subject is true is not the same as that of which the predicate is true. This is clear by the meaning of the negative copula.⁵⁰

⁴⁸ Marsilius, *Insolubilia*, ch.1 (S f.64r): "*Hiis descriptionibus positis sit prima suppositio hec: propositio que non est nec est vera nec falsa. Patet quia si est vera aut falsa sequitur quod est.*"

⁴⁹ Marsilius, *Insolubilia*, ch.1 (S f.67v): "*Secunda conclusio est: non potest esse totaliter sicut hec mentalis: omnis propositio est particularis, secundum eius totalem significationem tam materialem quam formalem significat, quia significat se esse per tertiam suppositionem et precedentem conclusionem [sc. "prima conclusio hec: omnis propositio categorica affirmativa significat se esse"], et hoc repugnat sue significationi materiali quod omnis propositio est particularis.*"

⁵⁰ Marsilius, *Insolubilia*, ch.1 (S f.67r): "*Quarta suppositio: omnis propositio categorica affirmativa affirmat subiectum et predicatum supponere pro eodem seu ponit idem esse pro quo accipiuntur extremitates. Patet per quid nominis copule affirmative ... Quinta suppositio: omnis categorica negativa significat subiectum supponere non pro illo pro quo predicatum, vel istud pro quo verificatur esse subiecti non verificari esse predicati de eodem. Patet per quid nominis copule negative.*"

It follows, he says, that every subject-predicate proposition has two significations, one about what is external to the proposition, its material signification, and another about itself, its formal signification:

It follows from [the fourth assumption] as a corollary that an affirmative subject-predicate proposition has two significations, namely, a material one about external reality (e.g., 'A man is an animal' signifies by this [type of] signification that a man in reality is an animal); and another formal one by which it signifies that the subject and predicate supposit for the same things (e.g., the aforesaid proposition signifies that 'man' and 'animal' supposit for the same things) ... [and from the fifth assumption] it follows as a corollary that every [negative subject-predicate] proposition has two significations, one about external reality, namely, the material one, and another formal one, namely, about itself and about its terms in this direct way corresponding to what was said about the affirmative proposition.⁵¹

Although, as Catarina Dutilh Novaes has described, Aristotle introduced hylomorphism into physics and metaphysics, he did not apply the formal-material distinction in logic.⁵² That was left for the medievals to do, and their first forays were to apply the distinction to the theory of supposition and the theory of consequence. But it was also applied in various ways in the theory of signification. Besides talk of the formal and material signification of terms,⁵³ many authors speak of the formal and material signification of propositions. However, rather surprisingly, there seem to be two contrary traditions making use of the latter distinction, one the inverse of the other. Buridan, for example, when describing his earlier position on the insolubles, writes:

⁵¹ Marsilius, *loc.cit.*: "*Ex quo sequitur correlarie quod affirmativa categorica duas habet significationes videlicet materialem de re ad extra: exemplum ut hec: homo est animal, hac significatione significat hominem ad extra esse animal. Et aliam formalem qua significat idem esse pro quo supponitur subiectum et predicatum ut propositio predicta significat idem esse pro quo supponit ly homo et ly animal ... Correlarie sequitur quod omnis talis habet duas significationes, unam de re extra videlicet materialem et aliam formalem scilicet de se et de suis terminis directo modo isto et proportionali ut dicitur de propositione affirmativa.*"

⁵² Catarina Dutilh Novaes, 'Form and Matter in Later Latin Medieval Logic: The Cases of *Suppositio* and *Consequentia*', *Journal of the History of Philosophy* 50 (2012), 339–64.

⁵³ See, e.g., Alfonso Maierù, *Terminologia Logica della tarda Scolastica* (Roma: Edizioni dell'Atenio, 1972), pp.111–12, 115–16.

Every proposition signifies itself to be true by its formal signification ... Some propositions signify themselves to be false by their material signification, that is, by reason of some terms occurring in them.⁵⁴

This use of the distinction is echoed by Albert of Saxony,⁵⁵ Marsilius,⁵⁶ Peter Tartaret (who retains 'material signification' to describe the sense generated by the terms, but uses 'reflexive signification' to describe the sense generated by the meaning of the copula)⁵⁷ and others. But later authors writing on insolubles in the fifteenth and sixteenth centuries invert this distinction. For example, George of Brussels writes:

Hence every insoluble has a two-fold signification, namely, a primary or direct signification which derives from the primary signification of the terms and not from the reflections of some part on the whole proposition. And this signification is usually called that which the terms primarily suggest, and by this signification it is meant that what the subject is taken for is the same as what the predicate is taken for, and among some it is usually called the formal signification. Another signification is secondary and indirect and it derives from the meaning of some extreme suppositing for the whole proposition of which it is an extreme, and by

⁵⁴ Buridan, *Questiones Elencorum*, § 19.3.2 (p.92): "*Et sit prima suppositio quod omnis propositio de significatione formali significat se esse veram ... Secunda suppositio quod aliqua propositiones de significatione earum materiali, puta ratione aliquorum terminorum in eis positorum, significant se esse falsas.*"

⁵⁵ Albert of Saxony, *Perutilis Logica*, ch.6: 'Insolubles' (ed. Berger, *Logik*, p.1124, trans. Kretzmann and Stump, p.348): "*Ista enim 'Haec propositio est falsa' de significatione formali, cum sit affirmativa, significat idem esse, pro quo supponunt eius subiectum et praedicatum, de significatione autem materiali, scilicet ratione istius termini falsa supponentis pro ipsamet propositione, ista propositio significat se esse falsam et per consequens non esse idem pro quo supponunt eius subiectum et praedicatum.*"

⁵⁶ Peter of Ailly also uses the terminology in his discussion of Marsilius in his *Insolubilia* (ed. Erne, 79; trans. Spade, §§ 247 ff. and notes 665 ff.).

⁵⁷ Peter Tartaret, *Expositio in Summulas Petri Hispani* (Freiburg im Breisgau: Kilianus Piscator, 1494), sig.k6rb–va: "*Sequitur de significatione propositionis insolubilis. Pro quo est advertendum quod significatio propositionis insolubilis est duplex. Quedam est materialis, que sibi convenit ratione impositionis terminorum, et hanc significationem primo intendunt termini ... Alia est significatio reflexa quam secundo per se dicunt termini: ut propositio ex reflexione eius supra seipsam in significando: et secundum istas duas significationes propositio dicitur insolubilis.*"

reason of this signification with the aid of the scenario the proposition is falsified; and this is usually called the material signification.⁵⁸

How such an inversion came about is puzzling and warrants further investigation. For now, however, we can take it that, at least according to Marsilius, the material signification of a proposition is its usual signification generated by its terms and the formal signification is generated by the meaning of the copula, affirmative or negative, that the subject and predicate do or do not, respectively, supposit for the same things. Together, these significations make up the total signification of a proposition.

Marsilius argues that these two significations must be combined conjunctively to yield the correct truth-conditions. The possible options are that they be combined either conjunctively, or disjunctively, or disjointly (*inconiuncte*). It can't be the last, because then the proposition would be merely ambiguous, that is, it would be a "*propositio plures*" (a manifold proposition), but that is not true in general:

For it cannot be said that it signifies them disjointly, because then every such proposition would be a manifold proposition and in need of disambiguation.⁵⁹

⁵⁸ *Expositio magistri Georgii Bruxellensis in logicam Aristotelis una cum magistri Thome Bricoti textu ...: liber primus Elenchorum* (Lugduni: Stephanus Gueynard, 1504), f.cclxxi: "Unde quodlibet insolubile duplicem habet significationem, sc. significationem primariam et directam que fit ex primaria significatione terminorum et non ex reflectiones alicuius partis supra totam propositionem. Et ista significatio solet vocari illa quam termini primarie pretendunt: et per hanc significationem denotatur quod illud pro quo accipitur subiectum est illud pro quo accipitur predicatum, et solet apud aliquos etiam vocari significatio formalis. Alia est significatio secundaria et indirecta et illa fit ratione alicuius extremi supponentis pro totali propositione cuius est extremum, et ratione illius significationis cum adiutorio casus falsificatur propositio et ista solet vocari significatio materialis." See E.J. Ashworth, *Language and Logic in the Post-Medieval Period* (Dordrecht: Reidel, 1974), pp.110–11. See also Jodocus Trutvetter, *Summule totius logice* (Erphurdie: Lupambulus Schenck, 1501), p.771: "Et itaque quodque insolubile falsum et duplicem complectitur significationem: primariam scilicet et secundariam. Primaria sive directa est que concrecit ex primaria terminorum significatione nec ex reflexione alicuius partis super totam propositionem: quam etiam vocant significationem quam termini primarie pretendunt: et per illam denotatur illud pro quo supponit subiectum esse id pro quo accipitur predicatum in propositione affirmativa. Nominatur ab aliis significatio formalis. Secundaria autem et indirecta sive reflexa est que fit ratione alicuius significationis cum adiutorio casus frequenter falsificatur propositio. Hanc alii significationem materialem appellant."

Nor can they be combined disjunctively, for if so, ‘Every proposition is false’, were it the only proposition, would be true. Since it is an insoluble and hence, as Marsilius will show later, is false, it follows that its subject and predicate supposit for the same, and thus things are as it signifies by its formal signification.⁶⁰

Consequently,

... an affirmative subject-predicate proposition connotes these senses conjunctively. This is clear because it signifies them (by the first conclusion⁶¹) and not disjointly (by the fifth [conclusion]) ... nor disjunctively (by the sixth) ... therefore conjunctively ... Similarly, a negative proposition connotes the two senses conjunctively. This is proved because just as an affirmative subject-predicate proposition is related to its affirmative significates, so a negative one to its negative significates. But an affirmative connotes its senses conjunctively, therefore, so too does a negative.⁶²

⁵⁹ Marsilius, *Insolubilia*, ch.1 (S f.68r–69r): “... enim dici non potest quod significet eas inconiuncte quia tunc quelibet talis esset propositio plures et distinguenda ... Quinta conclusio: propositio categorica huiusmodi non significat illos duos sensus inconiuncte. Patet quia significat eos naturaliter in mente, ergo coniuncte, quia si inconiuncte propositio mentalis esset plures et sic in mente esset equivocatio.” On the notion of *propositio plures*, see, e.g., Gabriel Nuchelmans, *Late-Scholastic and Humanist Theories of the Proposition* (Amsterdam: North-Holland, 1980), pp.119–24 and J.Spruyt, ‘The Forma-Materia Device in Thirteenth-Century Logic and Semantics’, *Vivarium* 41 (2003), 1–46, § 2.2. Nuchelmans conjectures that the curious term ‘propositio plures’ is short for ‘propositio plures sensus habens’.

⁶⁰ Marsilius, *Insolubilia*, ch.1 (S f.69r): “Sexta conclusio quod propositio categorica affirmativa significat sensus predictos copulative. Patet quia significat eos coniuncte et non significat eos disiunctive, cum non appareat aliqua nota disiunctionis. Et ex alio quia si eos disiunctive significaret hec esset vera: ‘omnis propositio est falsa’, illa sola existente in mundo cuius oppositum postea dicatur, et tenet consequentia quia valeret illam disiunctivam: ‘omnis propositio est falsa, vel subiectum et predicatum huius propositionis: “omnis propositio est falsa” supponit pro eodem’, restringendo a prima parte significationem formalem. Modo hec disiunctiva est vera, quare ipsa esset vera.”

⁶¹ The correct reference seems not to be to the first conclusion (“ponitur prima conclusio hec: omnis propositio categorica affirmativa significat se esse”), but rather to the corollaries to the fourth and fifth assumptions, cited in n.51 above.

⁶² Marsilius, *loc.cit.*: “Septima conclusio: quod categorica affirmativa istos sensus importat copulative. Patet quia significat eos per primam conclusionem et non inconiuncte per quintam nec disiunctive, per sextam, ergo copulative, quod est propositum. Octava conclusio: negativa similiter duos sensus importat copulative. Probatur quia sicut categorica affirmativa se habet ad sua significata affirmativa, ita negativa ad sua significata negativa. Sed affirmativa importat suos sensus copulative, ergo et negativa.”

4. Marsilius's Solution

Marsilius proceeds to apply this analysis to the truth-conditions of insolubles, that is, as he had defined them, self-falsifying propositions. By definition, they signify themselves to be false. But at the same time, he argues, they signify themselves not to be false. His argument is this:

Turning to the second chapter, it should be realised that if someone says about some utterance that it is false, the sense is that things are not as that utterance signifies by its primary representation. For if I said 'Socrates is running', and you said 'You say something false', the sense would be that things are not as my utterance signifies, and so quite generally in all cases when some utterance is said to be false, the sense is that things are not as that utterance suggests.⁶³

By the "primary representation", Marsilius means its material signification, what the utterance overtly says about how things are.⁶⁴ But after constantly using the terms 'material signification' and 'formal signification' in the first chapter, he only uses each of them a couple of times in the later chapters, preferring the terminology of 'direct signification' or 'direct sense' and 'reflexive signification' or 'reflexive sense'. A hypothesis, which needs further research, is that the second and subsequent chapters represent an earlier draft of the work, the first chapter

⁶³ Marsilius, *Insolubilia*, ch.2 (S f.71r): "*Quantum ad secundum tunc est notandum quod si aliquis de quocumque dicto dicat esse falsum, sensus est quod non est ita sicut istud dictum prima representatione significat. Si enim ego dicerem: sortes currit, et tu diceres: tu dicis falsum, sensus esset quod non esset ita sicut meum dictum significat, et sic est generaliter in omnibus quando de aliquo dicto dicitur esse falsum, sensus est quod non est ita sicut istud dictum pretendit.*"

⁶⁴ See his second conclusion, cited in n.65. See also Marsilius's later analyses of two sophisms, where he contrasts the primary signification (*prima significatio*) with the secondary, or reflexive signification, one in the fourth chapter (73v): (*Aliud sophisma: sortes dicit falsum, et ponitur quod istam dicat sortes et nullam aliam et non sit alius homo vocatus hoc nomine quam ipse ... sophisma est falsum, valet enim tantum sicut hec copulativa: sortes dicit falsum et falsum est sortem dicere falsum. Modo ista est falsa pro secunda parte. Et ideo licet semper ita sit sicut ipsa significat prima significatione, tamen non est ita sicut ipsa significat secunda significatione, scilicet reflexione falsitatis scilicet quod falsum sit sortem dicere falsum*); and one in the fifth (79r-v): (*Tertium sophisma: omnis propositio est similis propositioni p et vocetur hec: 'chymera est' nomine proprio p, et solum sophisma sit ... sophisma est falsum et etiam quod p est falsa et quod non sunt plures propositiones, et etiam est sicut sophisma significat prima significatione, et directa, sed sic non est sicut significat reflexa significatione, et ideo est falsum*).

constituting an uncompleted later revision which introduced the terminology of 'formal' and 'material' signification.

So take some self-falsifying proposition: what it says about itself is that things are not as it overtly signifies by its primary, or material, signification, that is, what it says is that it is not false, that is, it is not as it (primarily) signifies. Since it is self-falsifying, it says of itself that it is false. But in saying that it is false, it says that it is not as it signifies, that is, it says that it is not false.⁶⁵ So it is implicitly contradictory, in the tradition started by Bradwardine, in signifying both (overtly or primarily) that it is false and (covertly or reflexively) that it is not false. Hence any such proposition must be false, since things cannot be wholly as it signifies.⁶⁶

This covert, or reflexive, signification is what in the first chapter he called the formal signification. Being self-falsifying, such a proposition says of itself that it is false, that is, that the predicate 'false' or 'falsehood' co-supposits with a term referring to itself. Take 'Socrates says a falsehood', or rather, for clarity, making 'false' the predicate, 'What Socrates says is false'. Then 'false' and 'what Socrates says' co-supposit. But what Socrates says is that Socrates says a falsehood. Thus the formal, or reflexive, signification amounts to the claim that it is false that Socrates says a falsehood, that is, that Socrates does not say a falsehood. So the material, or direct, sense and the formal, or reflexive, sense are inconsistent and jointly unsatisfiable. Consequently, things cannot be wholly as 'Socrates says a falsehood' signifies. It follows that it must be false.

⁶⁵ Marsilius, ch.2 (S f.71r): "*Secunda conclusio est quod huiusmodi propositio significat se non esse falsam. Probo sic: ipsa dicit de se ipsa esse falsam, ergo per notabile significat de se non esse ita sicut significat prima representatione. Sed prima representatione significat se esse falsam, ergo significat non esse ita quod ipsa est falsa, ergo significat se non esse falsam.*"

⁶⁶ Marsilius, *Insolubilia*, ch.2 (S f.71v): "*Tertia conclusio: quod quevis huiusmodi propositio significat aliquo qualiter non est. Patet quia per primam conclusionem significat se esse falsam et per secundam significat se non esse falsam. Modo ille significationes sunt repugnantes per septimam suppositionem [sc. "eadem propositio non est vera et falsa"] quia non est possibile totaliter esse sicut propositio talis significat. Quarta conclusio: quod quevis huiusmodi propositio est falsa. Patet per precedentem conclusionem quia quevis significat aliquo qualiter non est.*"

In chapters 3–8 Marsilius applies this analysis to some thirty-odd sophisms. Take the fourth sophism from ch.4, for example, where he discusses the familiar scenario in which Socrates lays down that he ought to throw all and only those saying something false from the bridge, and not those speaking the truth, and Plato steps forward and says to Socrates that Socrates will throw him from the bridge.⁶⁷ Marsilius rehearses several arguments against his own claim that Plato’s utterance, being self-falsifying, is false. For in that scenario, those who say something false ought to be thrown from the bridge, so his utterance is true, whence by the law of contradictories (*lex contradictoriarum*) its contradictory, ‘Socrates will not throw Plato from the bridge’, is false. Marsilius replies that the contradictory of Plato’s utterance is not a subject-predicate proposition. For the signification of Plato’s utterance must be expounded by a three-membered conjunction, and so its contradictory must be a three-membered disjunction:

... the signification of [Plato’s utterance] ought to be expounded by a three-membered conjunction like this: Socrates will throw Plato from the bridge, and the proposition ‘Socrates will throw Plato from the bridge’ is false, and it is false that it is false. For that reason, its contradictory must be this disjunctive proposition: Socrates will not throw Plato from the bridge or the proposition ‘Socrates will throw Plato from the bridge’ is not false, or it is not false that it is false; and it is true on account of the last disjunct.⁶⁸

But why does Marsilius claim that three conjuncts are needed to express the total signification of Plato’s utterance? The reason would seem to be that, in this scenario, Plato’s utterance implies its own falsity even though it does not explicitly assert it. That is why Marsilius describes it as self-falsifying:

⁶⁷ Marsilius, *Insolubilia*, ch.4 (S f.73v): “... *sit casus quod sortes omnem dicentem sibi falsum debeat proicere de ponte et solum talem et dicentem verum non et adveniet plato et dicat sorti ‘sortes proiciet platonem de ponte’.*”

⁶⁸ Marsilius, *Insolubilia*, ch.4 (S f.74r): “... *contradictorium eius non est illa categorica: sortes non proicet platonem de ponte, sed eius significatio debet exponi per copulativa trimembrem sic: sortes proicet platonem de ponte et hec propositio: sortes proicet platonem de ponte, est falsa, et falsum est eam esse falsam, et ideo eius contradictoria debet esse hec disiunctiva: sortes non proicet platonem de ponte vel hec propositio: sortes proicet platonem de ponte non est falsa vel non est falsum quod ipsa est falsa; et illa pro ultima parte est vera.*”

For in this scenario this proposition falsifies itself because it signifies that Socrates ought to throw Plato from the bridge and according to the scenario that requires that it be false, so its truth requires that it is false, and for that reason it signifies itself to be false.⁶⁹

Thus Plato's utterance 'Socrates will throw Plato from the bridge' implicitly signifies its own falsity, and so the formal or reflexive signification is that 'false' co-supposits with a term specifying its primary sense, that is, that it is false that it is false. Accordingly, the proper contradictory of Plato's utterance has three disjuncts, the third of which is true, so making the whole disjunction true. Hence, by the law of contradictories, his utterance is indeed false, as Marsilius claims. What makes it false is not its primary representation (its material signification, namely, that Socrates will throw Plato from the bridge and that it is false) but its reflexive sense, its formal signification, that what it materially signifies is false, that is, that it is false both that Socrates will throw Plato from the bridge and that it is false.

Peter of Ailly claimed that this sophism is not an insoluble since it contains no term referring to a proposition, and dismissed Marsilius's account of insolubles for that reason. We noted earlier that, although Peter's definition of insolubles as propositions that signify themselves to be false bears a superficial resemblance to Marsilius's, Peter draws as a corollary from his definition the conclusion that any insoluble must contain "a term appropriately signifying a proposition, such as 'true', 'false', 'universal', 'partial', 'affirmative', 'negative', 'granted', 'denied', 'doubtful', 'certain', and such like." His reasoning turns on the claim that "a proposition does not signify except by reason of its terms."⁷⁰ We have seen that Marsilius would agree with this (see n.7 above: "not all difficult sophismatic propositions are included [among the insolubles], but only those whose difficulty

⁶⁹ Marsilius, *Insolubilia*, ch.4 (S f.73v–74r): "*Nam in hoc casu hec propositio falsificat se quia significat sortem platonem debere proicere de ponte et ad hoc ex casu requiritur quod ipsa sit falsa, ergo ad eius veritatem requiritur eam esse falsam, quare significat se esse falsam.*"

⁷⁰ Peter of Ailly, *Insolubilia* (ed. Erne, 77; trans. Spade, 65): "cum propositio non significet nisi ratione suorum terminorum."

[of solution] derives from the signification of the terms”). Nonetheless, Peter seems to overstate his claim here, for there is a very real sense in which Plato’s statement refers, albeit indirectly, to itself via Socrates’s announcement. That is the point of Marsilius’s observation that “the signification of [Plato’s utterance] ought to be expounded by a three-membered conjunction,” including the conjunct ‘the proposition “Socrates will throw Plato from the bridge” is false’.

A second example that is worthy of examination is the seventh sophism of ch.7. The subject of ch.7 is compound insolubles (*insolubilia hypothetica*), for example:

God exists and this conjunctive proposition is false (where this is the only conjunctive proposition in the world)

A man is an ass or some disjunctive proposition is false (where there is no other disjunctive proposition in the world besides this one, and let the disjunction be called A, its first part C and the second D)

If God exists, some conditional proposition is false (where this is the only conditional proposition in the world and no other, called by the proper name E, with premise F and conclusion G)

The conclusion of this consecution is false, therefore, the conclusion is false (and let there be only this consecution)

This consecution exists, therefore, this consecution is not valid (where by each occurrence of ‘this’ I refer to the consecution itself, and the consecution is called K, its premise L and its conclusion M).⁷¹

The extravagant naming of propositions and consecutions here is unusual.

Whereas most medieval authors rarely go beyond ‘A’ to name their sophisms,

⁷¹ Marsilius, *Insolubilia*, ch.7 (S ff.85r–87v): “... *primum sit copulativum talem: deus est et hec copulativa est falsa, et sit solum hec copulativa in mundo ... Secundum sophisma est tale: homo est asinus vel aliqua disiunctiva est falsa, et non sit aliqua disiunctiva in mundo quam hec. Et vocetur hec disiunctiva a, prima pars c et secunda d ... Tertium sophisma sit tale: si deus est, aliqua conditionalis est falsa, et sit solum hec conditionalis in mundo, et nulla alia, que vocetur proprio nomine eius e, et antecedens f et consequens eius g ... Sextum sophisma est tale: consequens illius consequentie est falsum, ergo consequens est falsum. Et sit solum ista consequentia ... Septimum sophisma est tale: ista consequentia est, ergo ista consequentia non valet, et demonstro per ly ista utrobique istamet consequentiam et vocetur ista consequentia k et antecedens eius l et consequens eius m.*”

occasionally introducing 'B' and 'C' only where absolutely necessary for distinguishing different elements within the same sophism, Marsilius proceeds right through the alphabet as far as 'P', 'Q' and 'R' ('P' to name a particular proposition said to be similar in truth-value to the sophism itself, which itself is subsequently named 'R').⁷²

The third of these compound sophisms, 'If God exists, some conditional proposition is false', is somewhat similar to another sophism, found in Albert of Saxony's *Insolubles* and other sources, which was rediscovered in the twentieth century by Haskell Curry, and has consequently become known as Curry's paradox, based on the conditional proposition 'If this conditional is true then God does not exist' (or '... then you are an ass', or '... then P' for an arbitrary proposition P).⁷³ Curry's paradox based on a conditional has been further extended to an inferential version, known as 'V-Curry', based on a one-premise consecution: 'This consecution is valid, so P', for some arbitrary proposition P.⁷⁴ Inverting V-Curry in turn yields (since P is arbitrary): 'P, so this consecution is not valid', of which Marsilius's seventh sophism (the fifth in the above list) is an instance, taking L, that is, 'This consecution exists', for 'P'. Marsilius's

⁷² Marsilius, *Insolubilia*, ch.5 (S f.79r): see n. 64. 'p' and 'q' are again used in the seventh sophism of ch.6 (S f.84v): "*Dicatur igitur quod si secunda pars disiunctive p [rex sedet vel disiunctiva scripta in hoc folio est platonis dubia] vocetur nomine proprio q tunc q falsificat se sicut patet ex dictis*"

⁷³ See H.B. Curry, 'On the inconsistency of certain formal logics', *Journal of Symbolic Logic* 7 (1942), 115–17; Albert of Saxony, *Perutilis Logica* ch.6 'Insolubilia', *Insoluble XIII* (ed. Berger, *Logik*, p.1154–8): "'Si deus est, aliqua condicionalis est falsa', et sit ista condicionalis in mundo et nulla alia," translated in *The Cambridge Translations of Medieval Philosophical Texts*, vol.1: *Logic and the Philosophy of Language*, 359–60.

⁷⁴ See, e.g., Lionel Shapiro and J.C. Beall, "Curry's Paradox", *The Stanford Encyclopedia of Philosophy* (Winter 2021 Edition), Edward N. Zalta (ed.): <https://plato.stanford.edu/archives/win2021/entries/curry-paradox/#ValiCurr>. V-Curry is already found in William Heytesbury's *Asinine Sophisms*, written in the early 1330s: see Fabienne Pironet, *Guillaume Heytesbury, 'Sophismata asinina', une introduction aux disputes médiévales. Présentation, édition critique et analyse* (Paris: Vrin, 1994), sophism 18, p.413: "*Tu es asinus. Probo: ista consequentia est bona: ergo tu es asinus, demonstrando per li 'ista' eandem consequentiam.*"

contraposed form of V-Curry is also famously found in Pseudo-Scotus's *Questions on the Prior Analytics*.⁷⁵

Suppose, therefore, that the consecution K in Marsilius's seventh sophism is sound—that is, valid (*bona*). Then, since things are as signified by its premise L ('This consecution exists'), it follows that things will be as the conclusion signifies, that is, K is not valid. So if K is sound, it is not valid. Conversely,

... if it is said that consecution K is not sound, on the contrary: it is impossible for things to be as the premise signifies unless they are as the conclusion signifies. Hence it is sound. The consecution holds by the definition of sound consecution and the premise is clear, because the conclusion signifies that consecution K is not valid. But if you say that it is not valid, it follows that whenever it exists it is not valid, therefore, whenever things are as the premise signifies they will be as the conclusion signifies. Therefore, consecution K is sound. So if K is not sound, it is sound.⁷⁶

Marsilius's response is to dismiss consecution K as invalid and to say that its conclusion is false, not on account of its primary or direct sense (that is, its material signification, that it is invalid) but on account of its reflexive signification. He accepts that the first part of the *reductio* argument, showing its invalidity, is sound:

⁷⁵ See Pseudo-Scotus, *Questiones super librum primum analyticorum*, in John Duns Scotus, *Opera Omnia*, ed. L. Wadding (Paris: Vivès, 1891–5), vol.2, 81–177, Q.10; translated in Mikko Yrjönsuuri (ed.), *Medieval Formal Logic: Obligations, Insolubles and Consequences* (Dordrecht: Kluwer, 2001), 225–34: see, in particular, pp.227–8.

⁷⁶ Marsilius, *Insolubilia*, ch.7 (S f.87v): "Tunc quero utrum k consequentia est bona. Et si dicitur quod sic, et notum est quod ita est sicut antecedens significat, ergo ita erit sicut consequens significat. Sed consequens significat k consequentiam non valere, ergo k consequentia non valet. Ergo si k consequentia est bona ergo k consequentia non valet. Si vero dicatur quod k consequentia non sit bona, contra: impossibile est sic esse sicut antecedens significat quin ita sit sicut consequens significat. Ergo est bona. Consequentia tenet per descriptionem consequentie bone et antecedens patet quia consequens significat k consequentiam non valere. Modo ex quo tu dicis quod non valet sequitur quod quandocumque ipsa est ipsa non valet, ergo quandocumque ita est sicut antecedens significat, ita erit sicut consequens significat. Ergo k consequentia est bona. Ergo si non sit bona, ipsa est bona."

I say that consecution K is not valid nor is it sound, because the conclusion falsifies itself. For from its being sound it follows that it is not sound, as the reasoning showed before the opposite [was claimed].⁷⁷

But the subsequent argument to the opposite, claiming that, assuming K is not sound, it follows that things can't be as its premise signifies unless they are as its conclusion signifies, ignores the reflexive signification of the latter:

For the conclusion implies this conjunctive proposition: consecution K is not sound and M is not false. But although the first part is true, nonetheless, the second part of this conjunction is false because M is not true. Indeed, the contradictory of the conclusion is true, namely, this disjunction: consecution K is sound or M is false.⁷⁸

The conjunction, 'consecution K is not sound and M is not false', is intended to spell out the total signification of M. The first conjunct clearly specifies the direct part of its signification (*conceditur ... consequens quoad significationem partialem et directam*). But we have already shown that M is self-falsifying in the first part of the argument, cited above. M entails that M is false and so falsifies itself. Hence:

The conclusion as regards its total and reflexive signification is not true.⁷⁹

Hence K is not sound and its conclusion is false.

5. Analysis

We noted that Peter of Ailly briefly discusses Marsilius's solution in his own treatise on insolubles, and dismisses it for failing to satisfy a corollary he draws

⁷⁷ Marsilius, *Insolubilia*, ch.7 (S f.88r): "Ad istud responditur breviter negando consequentiam et dicendo quod k consequentia non valet nec est bona quia consequens falsificat se. Ad hoc enim quod sit bona sequitur quod non sit bona sicut arguit ratio facta ante oppositum."

⁷⁸ Marsilius, *loc.cit.*: "Consequens enim valet hanc copulativam: k consequentia non est bona et m non est falsum, modo licet prima pars sit vera, tamen secunda pars istius copulative est falsa quia m non est verum, ymmo contradictorium consequentis est verum, scilicet hec disiunctiva: k consequentia est bona vel m est falsum."

⁷⁹ Marsilius, *loc.cit.*: "... consequens quoad significationem totalem et reflexam non est verum."

from his definition of 'insoluble'.⁸⁰ However, his argument for that corollary is unconvincing, as we saw in the last section.⁸¹ Paul Spade, in the 'Introduction' to his translation of Peter of Ailly's *Insolubles* (Spade 1980, p.6), likens Marsilius's diagnosis of the insolubles to Gregory of Rimini's, and Gregory's in turn to Heytesbury's. They are connected for Spade through the solution offered by Peter of Ailly, who seems to have known both Gregory's and Marsilius's discussions, though Gregory's is now known only through secondary sources, such as Peter of Ailly and Peter Tartaret.

I think the similarity to Gregory's solution is only superficial, and ostensibly misleading unless it can be backed by hard external evidence rather than mere resemblance. As reconstructed by Spade, Gregory's proposal rests on the claim, possibly prompted in reaction to Heytesbury's position, that insolubles cannot arise in mental language. For mental language, the language of the mind, has its signification naturally:

... for one of the features of mental language is that the signification of its [propositions] is fixed by nature once and for all, not by arbitrary conventions in the way the signification of spoken or written [propositions] is determined. Hence the signification of mental [propositions] cannot be "shifted" in the way Heytesbury requires of insolubles. In short, there can be no insolubles in mental language.⁸²

Spade backs up this claim about the fixity of mental signification by reference to passages in Gregory's *Sentences* and in Peter of Ailly's treatise, both expressing the popular Augustinian trope that thoughts are the same for all, signify naturally and belong to no (individual) language. But his argument is unconvincing, at least as a refutation of Marsilius's solution. A spoken or written term is imposed, by convention or at the pleasure of the speaker, on some thing or things as its significates; the concept which is the natural likeness of that thing or things is then automatically, or naturally, linked to that term. Nothing is changed or

⁸⁰ See n.15 and Spade, *Peter of Ailly: Concepts and Insolubles*, p.66 §247 (ed. Erne, pp.78–9).

⁸¹ See n. 69.

⁸² Spade, 'Introduction' to *Peter of Ailly: Concepts and Insolubles*, p.7. Spade uses 'sentence' for what I have been calling 'propositions'.

“shifted” according to Marsilius: that a proposition signifies (formally) that its subject and predicate supposit for the same is a necessary consequence of the meaning of the copula, regardless of whether the proposition is mental, spoken or written, or whether its material signification is gained by imposition or naturally.

Nonetheless, Spade conjectures, Gregory invoked this impossibility of mental propositions being insoluble to articulate a solution which claimed that every spoken or written insoluble of the form ‘S is P’ corresponds to a conjunctive mental proposition whose first conjunct expresses the immediate significate of the insoluble and whose second conjunct predicates ‘P’ of the first conjunct. For example:

... where *k* is the spoken or written insoluble [proposition] ‘*k* is false’, it corresponds to, or expresses, a mental conjunction the first conjunct of which is a non-insoluble mental [proposition] signifying only that the spoken or written insoluble *k* is false, and the second conjunct of which signifies only that the first conjunct is false.⁸³

Spade notes the similarity between Gregory’s mental conjunction of the form: ‘*k* is false and “*k* is false” is false’ and Marsilius’s conjunction of the material and formal significates of the insoluble. But their rationales for their accounts are very different. Take an arbitrary affirmative subject-predicate insoluble (spoken or written for Gregory, spoken, written or mental for Marsilius), of the form ‘*k* is false’, where ‘*k*’ reflexively supposits for ‘*k* is false’: then

‘ <i>k</i> is false’ is true iff	
Marsilius	Gregory
things are wholly as ‘ <i>k</i> is false’ signifies, ⁸⁴ iff	there corresponds to it a true strictly mental proposition, ⁸⁵ iff

⁸³ Spade, ‘Introduction’ to *Peter of Ailly: Concepts and Insolubles*, p.6.

⁸⁴ See the third definition, cited in n.47.

⁸⁵ Assuming, *faute de mieux*, that Peter of Ailly follows Gregory: see Peter’s *Insolubles* (ed. Erne, 32): “*Tertia conclusio est quod quaelibet propositio ad placitum significans ideo praecise est vera vel falsa quia sibi correspondet vera vel falsa propositio mentalis proprie dicta* (The third conclusion is that any proposition signifying by human

k is false and ‘that k is false’ and ‘false’ supposit for the same, ⁸⁶ iff	the mental proposition ‘ k is false and “ k is false” is false’ is true, iff
k is false and that k is false is false	k is false and the mental proposition ‘ k is false’ is false ⁸⁷

Either way, ‘ k is false’, for example, is false because the two conjuncts specifying its signification cannot both be true. In both cases, ‘ k is false’ is false because either k is false or it is false that k is false, and if it’s false it cannot be false that it’s false. But the reasons lying behind each author’s analysis are completely different.

As we have seen, however, the internal connection between Marsilius’s solution and the English tradition is much closer. For a start, three of the four alternative solutions which Marsilius rejects are from logicians at Oxford, or in Albert’s case, at the English Nation at Paris: the third opinion is that of Swyneshed, the fourth the modified form of Heytesbury’s solution found in Hunter, Holland and in many of the versions of the *Logica Oxoniensis*. The second is attributed to Buridan in an inter-lineal insertion by the scribe, though its description through the bare statement that it “claims that a self-falsifying

imposition is only true or false because there corresponds to it to a true or false strictly mental proposition). Spade, *Peter of Ailly: Concepts and Insolubles*, 46 has a slightly different translation.

⁸⁶ ‘ k is false’ signifies materially that k is false and signifies formally that ‘ k ’ (that is, ‘that k is false’) and ‘false’ supposit for the same, by the fifth conclusion of ch.2 (S 71v): “*Quinta conclusio sequitur ex primis duabus: quod oportet tales propositiones falsas et significationes earum explicari per unam copulativam, ut illius: ‘aliter est quam hec propositio significat, et aliter est quam quod aliter est sicut hec propositio significat’, vel loco secunde partis, ‘et non est aliter quam sicut hec propositio significat’. Similiter hec: ‘sortes dicit falsum’, exponitur sic: ‘sortes dicit falsum et falsum est quod sortes dicit falsum’, et ex hoc patet omnem talem esse falsam, quia quevis talis copulative est falsa.*”

⁸⁷ Again, assuming Peter of Ailly follows Gregory: see Peter of Ailly, *Insolubilia* (ed. Erne, 36): “... *quinta conclusio erit ista: Quod quaelibet propositio mentalis categorica proprie dicta simpliciter, de inesse et affirmativa, si sit vera, ideo est vera quia qualitercumque per eam secundum eius totalem significationem significatur esse, fuisse vel fore, taliter fuit, erit vel est* (... any strictly mental affirmative simply assertoric subject-predicate proposition, if it is true, is true because however things are signified by it to be, have been or will be, according to its total signification, in such a way they have been, will be or are).” Cf. Spade, *Peter of Ailly: Concepts and Insolubles*, 48.

proposition like ‘Socrates says a falsehood’ is false because it signifies itself to be true and false and that is not so” could apply to many proposed solutions, including those of Girard Odo and Thomas Bradwardine⁸⁸ as well as Albert of Saxony and Buridan. It also seems strange that an author writing in Paris in the 1360s, like Marsilius, should attribute such a view to Buridan, since the latter had clearly repudiated it in the 1350s, and also strange that it should be a conflation of Buridan’s early attribution of a metalinguistic signification to every proposition (which was the reason why Buridan rejected it) together with his later addition of the qualification ‘*virtualiter*’ to the implication of its own truth. But these alternative solutions may be Marsilius’s own invention, only based loosely on the actual views of his predecessors and contemporaries, purely for the purpose of exposition and motivation for his own solution.

6. Summary

Marsilius’s *Insolubilia* is preserved in three complete manuscripts and contains eight chapters, running to some 21000 words. In the first chapter, Marsilius sets out a number of definitions, postulates and conclusions. In the second, he first surveys four alternative opinions on the insolubles, which he rejects, before turning to his own solution. In the remaining chapters he shows how his preferred solution solves some thirty-odd sophisms. The central idea of his solution is that every proposition has two significations or senses, described in the first chapter as a material sense regarding something external to the proposition, and a formal sense resulting from the meaning (*quid nominis*) of the copula: if the copula of a proposition is affirmative, the formal sense signifies that the subject and predicate of the proposition supposit for the same thing or things; if the copula is negative, that the subject and predicate do not supposit for the same thing or things. In later chapters, these two senses or significations are called direct versus indirect or reflexive rather than material and formal. In the case of insolubles, these two senses conflict, for the material sense of the insoluble is, or

⁸⁸ On Odo and Bradwardine, see, e.g., Spade and Read, ‘Insolubles,’ *Stanford Encyclopedia of Philosophy*, § 3.8:
<https://plato.stanford.edu/entries/insolubles/#JohnBuri>.

at least implies, that the insoluble is false, whereas its reflexive, or formal, sense is that it is not false. So insolubles are false, since things cannot be in accord with their total signification.

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